[09-10-29-T-11] Rev 1

Basic trig

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[1] Find θ when $\mathbf{2\cos 3\theta = -\sqrt{3}}$

[2] Find θ when $\mathbf{\sin 2\theta = \frac{1}{2}}$ and $\mathbf{0 \le \theta \le 2\pi}$

[3] Solve for θ when $\mathbf{\frac{1}{3}\sin 2\theta = 1}$

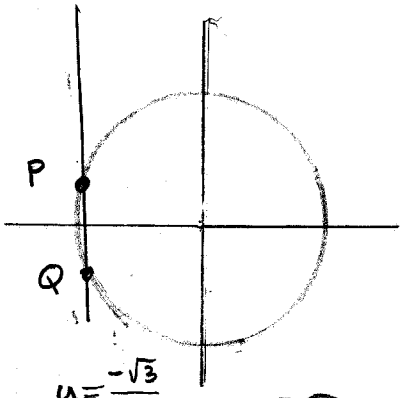
[4] Solve for θ when $\mathbf{\sin \frac{2}{3}\theta = \frac{1}{2}}$

[5] Solve for θ when $\mathbf{\sin 2\theta = 0}$

[6] Find θ when $\mathbf{\frac{\tan 2\theta}{\sqrt{3}} - 1 = 0}$

[1] Find θ when $2\cos 3\theta = -\sqrt{3}$

SOLN



$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$-\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

A $\quad \cos 3\theta = -\frac{\sqrt{3}}{2}$

P $\quad \cos\left(\pi - \frac{\pi}{6} + 2n\pi\right) = -\frac{\sqrt{3}}{2}$

Q $\quad \cos\left(\pi + \frac{\pi}{6} + 2n\pi\right) = -\frac{\sqrt{3}}{2}$

$(A, P) \Rightarrow \quad \cos 3\theta = \cos\left(\frac{5\pi}{6} + 2n\pi\right)$

$$3\theta = \frac{5\pi}{6} + 2n\pi$$

$$\theta = \frac{5\pi}{18} + \frac{2n\pi}{3}$$

$(A, Q) \Rightarrow \quad \cos 3\theta = \cos\left(\frac{7\pi}{6} + 2n\pi\right)$

$$3\theta = \frac{7\pi}{6} + 2n\pi$$

$$\theta = \frac{7\pi}{18} + \frac{2n\pi}{3}$$

$\therefore \theta \in \{x : x = \frac{5\pi}{18} + \frac{2n\pi}{3}, n \in \mathbb{Z}\} \cup \{x : x = \frac{7\pi}{18} + \frac{2n\pi}{3}, n \in \mathbb{Z}\}$

corrected

[2] Find θ when $\sin 2\theta = \frac{1}{2}$ and $0 \le \theta \le 2\pi$

SOLN

A $\quad \sin 2\theta = \frac{1}{2}$

P $\quad \sin\left(\frac{\pi}{6} + 2n\pi\right) = \frac{1}{2}$

Q $\quad \sin\left(\pi - \frac{\pi}{6} + 2n\pi\right) = \frac{1}{2}$

$(A, P) \Rightarrow \quad 2\theta = \frac{\pi}{6} + 2n\pi$

$\theta = \frac{\pi}{12} + n\pi$

$(A, Q) \Rightarrow \quad 2\theta = \frac{5\pi}{6} + 2n\pi$

$\theta = \frac{5\pi}{12} + n\pi$

	$\frac{\pi}{12} + n\pi$	$\frac{5\pi}{12} + n\pi$
$n = 0$	$\frac{\pi}{12}$	$\frac{5\pi}{12}$
$n = 1$	$\frac{13\pi}{12}$	$\frac{17\pi}{12}$
$n = 2$	$\frac{25\pi}{12}$	$\frac{29\pi}{12}$

Both greater than 2π

$\therefore \quad \theta \in \left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12} \right\}$

[3] Solve $\frac{1}{3}\sin 2\theta = 1$

SOLN

$$\frac{1}{3}\sin 2\theta = 1$$

$$\sin 2\theta = 3$$

$\therefore$ $\{\}$, because no number has a sine less than -1 or greater than 1; i.e.

$$-1 \leq \sin\phi \leq 1$$

[4] $\sin\left(\frac{2}{3}\theta\right) = \frac{1}{2}$

Soln

$$\sin\left(\frac{2}{3}\theta\right) = \frac{1}{2}$$

P $\quad \sin\left(\pi/6 + 2n\pi\right) = \frac{1}{2}$

Q $\quad \sin\left(\pi - \frac{\pi}{6} + 2n\pi\right) = \frac{1}{2}$

P $\quad \frac{\pi}{6} + 2n\pi = \frac{2}{3}\theta$

$$\theta = \frac{3}{2}\left(\frac{\pi}{6} + 2n\pi\right)$$

$$\theta = \frac{\pi}{4} + 3n\pi$$

Q $\quad \frac{5\pi}{6} + 2n\pi = \frac{2}{3}\theta$

$$\theta = \frac{3}{2}\left(\frac{5\pi}{6} + 2n\pi\right)$$

$$\theta = \frac{5}{4}\pi + 3n\pi$$

$\therefore \theta \in \{x : x = \frac{\pi}{4} + 3n\pi, n \in \mathbb{Z}\} \cup \{x : x = \frac{5\pi}{4} + 3n\pi, n \in \mathbb{Z}\}$

[5] $\sin 2\theta = 0$

SOLN

$$\sin 2\theta = 0$$
$$\sin(0 + 2n\pi) = 0$$
$$\sin(\pi + 2n\pi) = 0$$

$$2\theta = 0 + 2n\pi$$
$$\theta = n\pi$$

$$2\theta = \pi + 2n\pi$$
$$\theta = \frac{\pi}{2} + n\pi$$

$\therefore \theta \in \{x : x = n\pi, n \in \mathbb{Z}\} \cup \{x : x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}\}$

corrected

✓ [6] Find θ if $\dfrac{\tan 2\theta}{\sqrt{3}} - 1 = 0$

SOLN

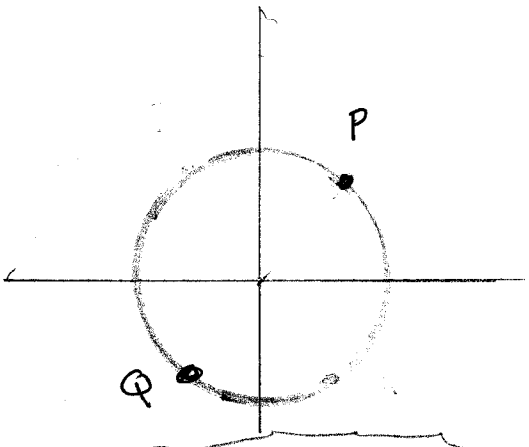
$$\frac{\tan 2\theta}{\sqrt{3}} + 1 = 0$$

$\equiv$ A $\quad \tan 2\theta = \sqrt{3}$

P $\quad \tan\left(\frac{\pi}{3} + n\pi\right) = \sqrt{3}$

Q $\quad \tan\left(\frac{\pi}{3} + \pi + n\pi\right) = \sqrt{3}$

(π: period of tangent function)



$\tan \frac{\pi}{3} = \sqrt{3}$

$\tan\left(\pi + \frac{\pi}{3}\right) = \sqrt{3}$

note that odd values of n will produce all points based on point P, and even values of n will produce all points based on point Q, so we only need A $\tan\left(\frac{\pi}{3} + n\pi\right)$

Thus,

$$\tan 2\theta = \tan\left(\frac{\pi}{3} + n\pi\right)$$

$$2\theta = \frac{\pi}{3} + n\pi$$

$$\theta = \frac{\pi}{6} + \frac{n\pi}{2}$$

$$\therefore \quad \theta \in \left\{ X : X = \frac{\pi}{6} + \frac{n\pi}{2},\ n \in \mathbb{Z} \right\}$$